DOCUMENT RESUME

ED 408 151 SE 059 973

AUTHOR Jaworski, Barbara

TITLE Tensions in Teachers' Conceptualisations of Mathematics and of Teaching.

PUB DATE Mar 97

NOTE 18p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, March, 1997).

PUB TYPE Reports - Research (143) -- Speeches/Meeting Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.

DESCRIPTORS Concept Formation; *Constructivism (Learning); Educational Strategies; *Epistemology; Foreign Countries; Knowledge Base for Teaching; *Knowledge Representation; Longitudinal Studies; *Mathematics Instruction; Secondary Education; Teacher Attitudes; *Theory Practice Relationship

IDENTIFIERS Great Britain

ABSTRACT

This paper addresses dilemmas in the practices of six teachers from the perspective of alternate paradigms of knowledge growth, particularly constructivism. The dilemmas focus on the relationships between theory and practice in the growth of knowledge in learning and teaching mathematics. The data for this study are drawn from a four-year classroom-based study of six teachers who chose an investigative approach to their teaching of mathematics at the secondary level. Highlighted in this paper are three worlds or discourses: (1) the academic world of contrasting theoretical positions aiming to inform the educational context; (2) the practical world of students' everyday lives, classroom relationships, and teaching decisions; and (3) the research world which tries to act as a bridge between the other discourses. The paper concludes with a brief discussion of negotiation as a dialogic mode which creates discourse from which individuals make their own sense and communicate with others. Contains 39 references. (DDR)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED 408 151

Paper associated with the presentation:

The Management of Learning in mathematics classrooms: tensions and issues from a constructivist perspective

In AERA Division C-4-1: Learning Environments Instructional Practices in Real and Virtual Classrooms

Chicago, March 1997

****

Tensions in Teachers' Conceptualisations of Mathematics and of Teaching

Barbara Jaworski - University of Oxford, UK.

For further discussion of this paper, please contact:

Dr. Barbara Jaworski
Centre for Mathematics Education Research
University of Oxford
Department of Educational Studies
15 Norham Gardens
Oxford OX2 6PY, UK

Phone:+44 1865 274024
Fax +44 1865 274027
E-mail: barbara.jaworski@edstud.ox.ac.uk

The paper is due to be published as a chapter in the book:
Learning Mathematics: From Hierarchies to Networks,
edited by Professor Leone Burton at the University of Birmingham, UK

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

B. Jaworski

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

SE059973

BEST COPY AVAILABLE

> I feel in my head I have a system of mathematics. I don't know what it looks like but it's there, and whenever I learn a new bit of mathematics I have to find somewhere that that fits in. It might not just fit in one place, it might actually connect up a lot of places as well. When I share things it's very difficult because I can't actually share my mathematical model or whatever you want to call it, because that's special to me. It's special to me because of my experiences. So, I suppose I'm not a giver of knowledge because I like to let people fit their knowledge into their model because only then does it make sense to them. Maybe that's why if you actually say, 'Well probability is easy. It's just this over this.', it doesn't make sense because it's got nowhere to fit. That's what I feel didactic teaching is a lot about, isn't it? Giving this knowledge, sharing your knowledge with people, which is not possible? (Ben, in Jaworski, 1994, p 157)

These are the spontaneous words of a teacher, Ben, talking to a researcher, myself, between lessons at which I was present as participant observer. Over a period of 9-12 months I explored with Ben his beliefs and theories, the motivations guiding his mathematics teaching and their rationalisation with his classroom practice. This took place as part of a 4-year classroom-based study of six teachers who chose an *investigative* approach to their teaching of mathematics at secondary level.

This paper addresses dilemmas in the practice of the six teachers from a perspective of alternative paradigms of knowledge growth, particularly constructivism, and, through this, relationships between theory and practice in the growth of knowledge about learning and teaching mathematics.

Interpretation and Construction

It is important to recognise that this research was *interpretive* at many levels. The teachers in the study interpreted an investigative approach in their classrooms. This involved interpretations of teachers' own theoretical perspectives as well as the various social/cultural worlds impinging on their classrooms: school and classroom culture, the culture of mathematics, the demands of schooling and assessment, home, family and ethnic groupings, and so on. As a researcher, presenting what I saw to occur, I offer interpretations arising from my own perspectives. Many of these were fed back to the teachers for their comments. However, since they are written from my own theoretical positions and research paradigms in which the research is embedded, they are often not commensurate with the teacher's perspectives. I therefore need to recognise the total situation and context of analytical outcomes, including research(er) decisions, to ensure rigour in the research. In terms of a currently popular paradigm (e.g. Bruner, 1986; Burton, 1996), these alternative interpretations might be seen as *narratives* which jointly illuminate knowledge growth for students, teachers and researchers relative to their domain of experience.

The research was embedded in a constructivist view of knowledge and learning, initially a *radical* constructivist view, and later one which is better described as *social constructivist*. This means that I interpreted events through my own constructivist perspective, and that an important part of my synthesis involved a justification of interpretations relative to this position. It does, however, make problematic any interpretation from an alternative paradigm.

There was no intent in this research to label the teachers as constructivist. Indeed, I question whether such a label has any meaning at all, since constructivism is not about

pedagogy. In fact, constructivism was never mentioned between us during the classroom study. Despite this, I argue that teachers could be *seen* as developing their teaching from a constructivist perspective. One teacher, reading my work much later, commented "so I was constructivist before I knew what one was. Does that mean I constructed constructivism?" (Mike, in Jaworski, 1994, p 132). The quotation from Ben, with which this paper begins, is paradigmatic of the position of teachers in this study. Analysis of this position will be left until after some discussion of the theoretical perspectives involved.

In order to explain, illuminate and critically situate issues arising from classroom situations, some rationalisation relative to theories of the growth of knowledge is essential. This is not to reduce the classroom richness and complexity to exemplification of particular theoretical perspectives but rather to engage in a dialectical relationship between given theories and interpretations of practice. Validity in interpretive research leads to a positing of theoretical perspectives which unsurprisingly intersects with theories in the public domain. John Shotter (1995), in a critique of a number of theoretical positions, questions relationships between theory and practice:

> If practice is not learned by first learning theory – and theory is not merely an accurate representation of a state of affairs – then what do we academics have to say that is of any worth to practitioners? And if we do have anything, how best should it be communicated to them if not as a theoretical representation?

My perspective here is to use theory in the public domain, together with its associated questions, issues and dichotomies, as a lens or lenses into practice, to, reflexively, inform and critique perceptions of knowledge and its growth from both theoretical and practical positions. Inevitably of course the discourse remains theoretical, and its relationship to its practical manifestations a matter of philosophical distinction.

Theoretical perspectives

RADICAL CONSTRUCTIVISM

Throughout the 1980s constructivism in mathematics education developed in a very theoretical way. Largely through the writings of Ernst von Glasersfeld, drawing strongly on the work of Piaget, a philosophy, epistemology or ideology, relating to mathematical cognition and cognitive processing was promulgated. Von Glasersfeld, supported by other scholars, presented a persuasive view of 'coming to know' in mathematics, which he referred to as Radical Constructivism. His definition of two principles is now very well known:

> 1 Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognising subject.
>
> 2 a. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;
>
> b. Cognition serves the subjects' organisation of the experiential world, not the discovery of an objective ontological reality.
> (von Glasersfeld, 1990)

It is the second of these principles which is *radical* in that it breaks away from a traditional metaphysical epistemology. It requires a recognition of the adaptive nature of cognition and the relative position of knowledge. From a radical constructivist perspective it is impossible to talk about the status of knowledge in absolute terms. Rather, knowledge needs to be related to its (historical) situation and context which often means that of the individual knower. Contrary to some criticisms of radical constructivism , this position does not reduce

to solipsism (Lerman 1989, 1996: Gergen, 1995). The very recognition of the relative nature of knowledge forces a critical rationalisation with experience, as well as with socially constituted bodies of knowledge and other knowers (which has profound implications for research). Kenneth Gergen's claim (1995, p28) that this implies "that there *is* a real world that is separate from one's experiences of it, thus reasserting the dualist assumption" is not one which I accept. Radical constructivism says no more than *if* there is such a world we cannot know it except through our experiences.

The profound implications for research in a constructivist paradigm are highlighted by Martin Hammersley, who, referring to the 'cultural relativism of constructivism', suggests it is unclear how constructivism differs from 'fiction or ideology', in that "research reports ... cannot be judged in terms of their validity, in the sense of how accurately they represent the events of the world ..." (Hammersley, 1993). Essentially, what is problematic with this statement is its assumption that validity rests with "how accurately [research reports] represent the events of the world", since it assumes we can have objective knowledge of these events. In a constructivist paradigm, validity must be related to a making sense of judgments in terms of their full situation and context. Other paradigms take a related stance, for example the *narrative paradigm* mentioned earlier and *the discipline of noticing* articulated by John Mason[1] . Hammersley, however, speaks from an objectivist paradigm incommensurable with constructivism since it depends on acceptance of ontological reality beyond the experience of the knower. It is worth drawing attention to this as relevant to the validity of the research in focus, but also since the issue is at the root of teaching dilemmas which are discussed later in this paper.

THE SOCIAL DIMENSION

The social constitution of knowledge is central to a *social* constructivist orientation. This orientation is not an 'add-on' to a radical position, but rather the result of placing a magnifying lens onto certain aspects of human experience which foster rationalisation, i.e. interactions and communication between human beings. To emphasise the importance of the social perspective, Taylor and Campbell-Williams (1993) have offered a 'third principle' to the two from von Glasersfeld.

> The third principle derives from the sociology of knowledge, and acknowledges that reality is constructed intersubjectively, that is it is socially negotiated between significant others who are able to share meanings and social perspectives of a common *lifeworld* (Berger and Luckmann, 1966). This principle acknowledges the sociocultural and socioemotional contexts of learning, highlights the central role of language in learning, and identifies the learner as an interactive co-constructor of knowledge.

While this articulation of the social position is helpful, it must be recognised that it is not an 'extra' to the other two, but rather a qualification of the second. It emphasises the importance of sociocultural settings in influencing cognition, and of cooperation and negotiation with others in offering alternative perspectives and challenging constructions. What the authors do not do, in this paper, is account for intersubjective construction of reality, and the roles of sociocultural and socioemotional contexts in knowledge growth. We need to turn to a debate about Piagetian and Vygotskian developmental frames to see why this might be problematic.

---

1 Briefly, noticing significant fragments of experience, accounting for their significance, and seeking resonance with others is the basis of a paradigm through which knowledge can grow within a community of practitioners. (e.g. Mason 1990, 1994)

THE INDIVIDUAL AND THE SOCIAL: PIAGET AND VYGOTSKY

While radical constructivism can be seen to develop from the work of Jean Piaget (e.g. von Glasersfeld, 1982), particularly from the point of view of accommodation of experience and reflective abstraction of concepts, social constructivism has been aligned with Vygotskian theory (Ernest, 1991). Lev Vygotsky emphasised the fundamental role of social influences on learning, particularly the role of language. Thus, it might seem seductive to seek a conjunction between aspects of Piagetian and Vygotskian world views to elaborate the social dimension of constructivism. However, it has been pointed out that this leads to an inconsistent absorption of Piaget into Vygotsky or vice versa (Confrey, 1995; Lerman, 1996).

Vygotsky wrote that "Human learning presupposes a special social nature by which children grow into the intellectual life of those around them" (1978, p 88). This might seem in striking contrast to a Piagetian view (in Jerome Bruner's terms "a paradigm of a lone organism pitted against nature" 1985, p25) focusing largely on individual cognition and developing the well known and much criticised stage theory. Yet Piaget not only recognised the importance of the social domain, he articulated explicitly its position with regard to his theory of intellectual development (Piaget, 1950, p156ff). According to Piaget, the social environment affects human beings "just as much" as their physical environment. He concedes that "society, even more than, in a sense, the physical environment, changes the very structure of the individual". He goes on to say that ways in which society interacts with an individual's consciousness, although potentially more fruitful, are not materially different from interactions with the physical world.

On the other hand, language (signs), the content of interaction (intellectual values) and rules imposed on thought (collective logical or pre-logical norms) "enrich and transform the individual's thought". Piaget suggests that cooperation, "a reciprocity between individuals who know how to differentiate their viewpoints", is particularly important to the constitution and development of logic; the decentering required for accommodation to signs, intellectual values, and logical norms in cooperative relationships parallels grouping of operations which signify intellectual development. He raises an issue which seems central to commensurability with Vygotskian theory:

> ... does operational development within the individual enable him to cooperate with others, or does external cooperation, later internalised, in the individual, compel him to group his actions in operational systems? (Op cit, p163)

Much quoted words from Vygotsky seem to support the second of these interpretations:

> Any function of the child's social development appears twice or on two planes. First it appears on the social plane and then on the psychological plane. First it appears between people as an interpsychological category and then within the child as an intrapsychological category. (Vygotsky, 1981, p63)

Piaget's further discussion of the issue suggests a necessary complementarity between the two positions, the "equilibrium of inter-individual interaction and that of the operations of which every socialised individual is capable when he reasons internally in terms of his most personal and original ideas". In fact the language in which Piaget expresses ideas (and I recognise the potentially distorting effects of translation into English) is wholly towards the autonomy of the individual developing consciousness, albeit influenced significantly through social interaction. Radical constructivism sits comfortably with these ideas, the social dimension being no more than an important part of human experience. Problems arise,

however, in accounting for any intersubjectivity of knowledge growth within a social dimension; more so in accepting the social dimension as pre-eminent. Social constructivism has to attend to these issues.

Interpretations of Vygotsky's writings, for example the quotation above, and more especially, "the social dimension of consciousness is primary in time and in fact. The individual dimension of consciousness is derivative and secondary" (Vygotsky, 1979, quoted in Wertsch and Toma, 1995) have led to a *sociocultural* view of the growth of knowledge in which the social domain is pre-eminent, and through which all knowledge is seen to develop. For example, James Wertsch (1991) speaks of a "sociocultural approach to mediated action", and Wertsch and Toma (1995 p 159) write "A fundamental claim of this approach is that mental functioning is assumed to be inherently situated with regard to cultural, historical and institutional contexts." From this perspective, *individual* functioning, if given credence at all, is seen as derivative of social or cultural knowledge. The authors state that "... social processes are given analytical priority when understanding individual mental functioning, rather than the other way round".

Sociocultural theorists such as Jean Lave & Etienne Wenger (1991) go further to speak of the development of communities of practice within which novices develop as full members of the community through apprentice-type relationships. Learning is seen to be a process of enculturation where learners as 'peripheral participants' in the community grow into 'old stagers', those who represent the community of practice. The authors write "... newcomers' legitimate peripherality ... involves participation as a way of learning – of both absorbing and being absorbed in – the 'culture of practice' ". They suggest that "mastery resides not in the master, but in the organisation of the community of practice". Thus, knowing, or cognition, is *situated* in the practice. This position might be seen as an interpretation of the Vygotskian image of children growing into the intellectual life of those around them. It is as if the community takes on the role of reality. Reality is interpreted through the norms and practices of the community.

Thus we see a conflict between metaphors of construction and enculturation which is not easily resolved. Confrey (1995) has suggested that an integrated theory is desirable, but difficult to achieve. Her review (Confrey, 1991) of a recent translation of Davidov's work on *Dialectical Materialism* which is situated in Russian *activity theory* indicates that activity theory offers a potential middle ground. Davidov claimed to have achieved an advance over both Piaget and Vygotsky, using their strengths but overcoming their weaknesses. These weaknesses, according to Confrey, lie in a) Piaget's treating of people and objects as equally significant members of a child's environment without significant discussion of the role of cultural transformation of knowledge, thus placing considerable demands on the creative and problem-solving ability of the individual child; b) Vygotsky's silence on how one builds up an awareness of objects through the manipulation of them. Davidov's work includes both emphasis on the role of cultural transformation of knowledge and a detailed attention to how one builds up an awareness of objects through their manipulation. Its limitations, Confrey suggests, lie in assumptions that cultural transformations are uniform across settings providing access to true reality and failing to provide a role for criticism and debate.

Confrey has herself suggested a weaving of theoretical perspectives to produce an alternative theory to Vygotsky and Piaget in which "individual and social development shape each other, [with] an appropriate balance of each" (1995, p225). She suggests that a critical

dimension of a new theory would be "the construction of self ... to allow for multiple selves out of which one forges one's identity." She concludes:

> I predict that the new theory will establish a relatively distinct basis – one in which diversity plays a more significant role, and in which the individuality of the child is tempered by the responsibility of community and culture. (op cit)

PROBLEMS IN RATIONALISING THEORIES: RELATION TO PRACTICE

It is in this complex theoretical arena, of which I have barely scratched the surface, that the classroom research in which I engaged is situated. Theories, I suggest, are sterile without rationalisation with practice. However, too often, the only rationalisation which occurs is speculative as theorists *suggest* outcomes or implications for theoretical manifestation in practice, or make limited investigations. What is actually needed is parallel explorations in both theory and practice, so that a genuine theory-practice dialectic might result. Stephen Lerman has acknowledged this in his consideration of research in the area of teachers' beliefs and practice:

> What constitutes a case of something that is specified in the theory-about-practice may well look different to a theory-building observer. This raises some serious questions about the whole nature of research on teachers' beliefs about their practices ... . Many of us, and I include myself in particular, have tried to theorise about teachers' beliefs about their practices by interviews in a different setting to the place in which the practice occurs. That approach has to be seen as extremely problematic." (Lerman, 1996a)

Where constructivist theory is concerned, the 1990s have seen a shift in focus towards theory-practice rationalisation through reporting of classroom research (e.g. the studies reported in Davis *et al*, 1990). More is needed, however, in bringing the issues from classrooms to centre stage, to challenge and potentially illuminate theory. Among questions needing considerably more attention and focused research, the following are central:

- what experiences are essential to learners in coming to know and be fluent with mathematics?
- what essential roles must a teacher play in fostering mathematical learning and what are the issues which arise?

In addressing these questions from a constructivist (or any other) perspective we must look critically both at the implications of the theory for learning and teaching in classrooms, and at the development or refining of theory as a result of rigorous research. Numerous research studies offer insights: for example, the work of Bauersfeld, Voigt, Steinbring *et al* in Germany (e.g. Bauersfeld, 1994) has clarified classroom interaction through microanalysis of classroom transcripts and a critical view of classroom knowledge as taken-as-shared; Cobb, Wood and Yackel, in the USA (e.g. Cobb *et al*, 1991) have conducted teaching experiments with whole classes of pupils, looking critically at negotiation between pupils and overt critiquing by pupils of each other's methods. In my own work I have looked at teachers' constructions of the processes of teaching mathematics, and in particular at the tensions which derive from a constructivist perspective on learning. Such studies take considerable time as processes emerge and methodologies develop. It seems crucial therefore, that the resulting insights, substantive and methodological, inform both theory and subsequent practice. I intend to address consequences of this for certain insights which emerged from my own research, and then return to theoretical issues outlined above.

Sources of tension in mathematics teaching

INTERPRETATIONS OF CLASSROOM EVENTS

A teacher had set up a series of lessons on *tessellation*, in which she wanted students to investigate aspects of tessellation and to mathematise[2] their findings, and from which she hoped her students would learn certain 'facts'. One of these facts was that 'all quadrilaterals tessellate', because their angle sum is 360 degrees. The tessellation can be achieved by placing four different angles of four quadrilaterals together at a point, and many students had noticed this as part of their exploration. However, the teacher, reflecting on her lesson, said:

> They [the students] kept referring to the fact that if they were able to make the shapes into quadrilaterals or rectangles, that they would be able to tessellate the shapes. But, yet, they weren't convinced that all quadrilaterals tessellated. That was the thing I wanted them to go on to ... (Felicity in Jaworski, 1994 p85)

While recognising valuable exploration and discussion of properties of shape, she emphasised the difficulty of getting students to come to the particular mathematical facts she wanted them to know. She did not wish just to tell them the fact, feeling that this was inappropriate to their investigative process. She recognised, therefore, a problem in the approach they were developing. In discussion with another teacher, they acknowledged their students' perspective:

J I think they would like to be told exactly what to do.

F I think they would lap it up.

J I think that's what they've been used to. I don't think they like it how we're actually doing it now, when we're actually making them think, and making them trying to figure it out for themselves. I do really think they hate that.

F It's expectation isn't it?

J Being brought up to expect to be told.

(Jane & Felicity in Jaworski, 1994, p 84)

It seemed important to the teacher that the students should come to know that 'all quadrilaterals tessellate', but that telling them would be ineffective for their learning. The students, however, would prefer to be told. Both teachers' and students' epistemological positions were a barrier to 'how we're actually doing it now'. The dichotomy here lies in conceptualising the 'it', both in terms of the ontological status of the mathematical 'fact', and in terms of how such facts come to be known. The teachers wanted to encourage students' autonomy in knowledge construction, but the construction had to result in particular knowledge.

Noddings (1990) asks what sort of assumption is being made when one says, 'All knowledge is constructed'. She writes:

> First ... Given a statement offered as a bit of knowledge, how does the claim about construction help us to decide what becomes part of the bona fide body of knowledge and what does not? Second, if we focus on knowers, how do we judge when they know and when they do not?
> (Noddings, 1990)

---

[2] I follow David Wheeler (1982) in using this term to mean bringing mathematics into being from "situaions where something not obviously mathematical is being converted into somthing that most obviously is".

These questions seem central to the teacher's perspective in the above discussion, and I would add a third question:

> Given a statement which *is* regarded as part of a bona fide body of knowledge, how does a learner *come to know* it and what does knowing it mean?

Moreover, for the teacher, there is also the question of what teaching acts will promote such knowing. In the series of lessons on tessellation, one teacher set up an activity based on the notion of tiling her kitchen. Students were asked to explore whether certain quadrilateral shapes would produce a tiling. Half way through the whole class discussion, one boy asked "Miss, why don't you have a carpet in your kitchen?" At a time when discussion had moved into a mathematical realm, where the kitchen idea had seemingly been left behind, this student's attention was on the everyday domain in which the topic had been introduced, and it seemed that the mathematical discussion lacked relevance or interest.

SOCIOCULTURAL INFLUENCES

Some observations about sociocultural influences in the above situations seem worthwhile before going further:

1. Teachers work with a given curriculum whose assessment is a strong influence on classroom norms ("what they've been used to"). Its (perceived) ontological status seemed to be at odds with teachers' desire for students' autonomy of mathematisation. Here we see institutional influences counteracting teachers' preferred approaches to mathematical learning.
2. In her words that students were "not convinced that" quadrilaterals tessellated, the teacher hinted at the domain of justification and proof in mathematical generalisation. This mathematical culture was not one with which students were comfortable, and this led to puzzlement about a need for universal statements.
3. The (pseudo?) positioning of the mathematics in the everyday world of students led to an issue for at least one student who (seriously or jokingly) implied that there was a simpler solution if indeed the discussion was about kitchen floors.

Students' conceptualisation is undoubtedly influenced by these intersecting cultural domains: to what extent and in what way they affect observed outcomes begs further study. Such influences contribute to the range of issues which have to be faced by teachers in constructing the classroom environment. The teachers' own 'coming to know' involves local rationalisation of these issues. Their critical reflection on the outcomes of local decision-making leads to explication of teaching processes. An important outcome of the research was the insights it provided into teachers' developing thinking and the concomitant development of their teaching. Evidence of this appears in the next sections.

QUESTIONING TEACHING DECISIONS

The teachers in this study engaged in a level of questioning of their local decisions and actions which led to a more conscious awareness of teaching processes. For example, the issue of 'when to tell':

> I'm conscious often of having at the back of my mind the desire not to tell an answer, and I will often ask so many questions that in the end I have more or less said "what is 2 and 2" just to get them to say a word. Because you feel that once they have said an answer then that is it. I'm conscious of that at the back of my mind, but I don't

> think there is anything wrong in sometimes admitting they've reached a stage where I've got to tell them something. (Mike in Jaworski, 1994, p120)

This quotation provides insight to the teacher's struggle with relating the act of teaching to a student's conceptualisation. Yet another teacher addressed similar issues from the perspective of her own knowledge:

> The way I work with these things [investigations] is that if I know too much about where it's going, given that I do prod and guide, I may well prod and guide people into directions which may not be the most fruitful ones, may not be the most interesting ones for them. ... Vicky and Ann were working in a way which I thought was not very fruitful ... I haven't prodded them very much, I haven't guided them very much, and the fact that Ann said a few things earlier on in this lesson helped actually, because I was able to say 'what was your idea?', 'what did you think you should do?' ... after all, I'm supposed to be a teacher and sometimes I do know that some ways are more fruitful than others, but only ... oh dear, it's terribly difficult isn't it.
> (Clare in Jaworski, 1994 p 137)

The research study provided a communicative domain in which questions would be addressed and issues acknowledged. It was clear that the role of the researcher was important to this process in a number of ways, including posing questions and listening to teachers' articulations and sometimes engaging in debate with the teacher. The teacher-researcher relationship provided a medium for sharing and negotiation of teaching issues. I suggest that a growth of knowledge in teaching stems from such articulations by teachers of issues in their practice, and a supportive community aids this process.

The researcher is also in the process of constructing knowledge, while making sense of situations and teachers' articulations relative to wider knowledge and theoretical positions. Although I quote the teachers accurately, my use of their words and the story I tell is my own construction from these events, albeit checked against teachers' own perspectives and offered for resonance in the educational community more widely – again, the supportive community. The next three sections offer a researcher perspective, or analysis, of events.

TO INCULCATE OR TO ELICIT?

The dilemma which I see voiced in these teachers' statements concerns interaction between students and the teacher with regard to the construction of knowledge. The student's task is to construct mathematical knowledge. The teacher's task is to support and challenge this construction. This is easy to say, but what does it mean? Edwards and Mercer (1987) articulate the dilemma aptly when they describe teachers as having to "inculcate knowledge while apparently eliciting it". *The teacher's dilemma*, as they call it, lies in "the problem of reconciling experiential, pupil-centred learning with the requirement that pupils rediscover what they are supposed to". It might also be described as a conflicting intersection of two paradigms: an objectivist paradigm in which the required curriculum and its examination structures are based, and a constructivist paradigm in which the teaching is situated. It might also be described socioculturally as the intersection of two irreconcilable cultures.

Driver (1983), writing of science teaching, spoke of 'intellectual dishonesty' in teaching:

> Secondary school pupils are quick to recognise the rules of the game when they ask 'Is this what was supposed to happen?' or 'Have I got the right answer?'. The intellectual dishonesty of the approach derives from expecting two outcomes from pupils' laboratory activities which are possibly incompatible. On the one hand pupils are expected to explore a phenomenon for themselves, collect data and make

> inferences based on it; on the other hand this process is expected to lead to the currently accepted law or principle. Driver (1983)

Since 'all quadrilaterals tessellate' can be proved using commonly agreed mathematical logic, is it more therefore than a currently accepted principle? And what does it mean to *know* it? The integrity of teaching seems related to answers to these questions. What are the perceived relationships between knowledge and knowing, telling or not telling, prodding and guiding? Although we might understand and appreciate the term, 'intellectual dishonesty' is a harsh phrase. Perhaps the teachers' imputations of guilt ("I don't think there's anything wrong in ..." and "after all I'm supposed to be a teacher ...") are an emotional response to feelings of injustice to students while in the grip of forces difficult to resist. Teaching dilemmas involve coping with moral and emotional issues whose resolution is far from clear.

DIDACTIC OR INVESTIGATIVE?

In characterising an *investigative* approach to teaching mathematics, it became clear that teachers saw it in contrast to, and to be preferred over, a so-called *didactic* approach. The teacher, Ben, quoted at the beginning of this paper, suggested that 'didactic teaching' is about giving knowledge; in terms expressed above, *telling* or *inculcating*. He had an explicit objective to implement an investigative approach to his teaching. In some of the lessons I observed, he seemed reasonably satisfied that he had achieved this but, in other lessons, he was critical that his approach had been, or would be, "more didactic than usual". In exploring the differences he perceived between investigative and didactic approaches, I came closer to understanding sources of tension.

Ben's use of the term 'fit' for making sense of a concept relative to one's experience seemed to accord strongly with von Glasersfeld's articulation of radical constructivism (e.g. 1994, p21). Von Glasersfeld has written "The teacher will realise that knowledge cannot be transferred to the student by linguistic communication but that language can be used as a tool in the process of guiding the student's construction" (1987). Ben, along with other teachers, seemed to agree almost literally with the first part of this statement, but without having a clear rationalisation of the second part. Here is an example where links between theory and practice seem particularly fragile. What exactly does it mean to use language 'as a tool in the process of guiding the student's construction'? The practical manifestation of such an idea is problematic, an issue for concern. All of the teachers quoted show evidence of struggling with this issue.

Lessons which were regarded as investigative were based on enquiry and questioning. Students were expected to explore a given situation and derive their own mathematical formulations. For example, in one lesson Ben asked students what shapes they could find whose area and perimeter were numerically the same. (He called them *Kathy* shapes.) This led to consideration of properties of shape, of the relationship between area and perimeter of shapes, and development of methods of trial and improvement. There was evidence of students' high level mathematical thinking (conjecturing, generalising, critical questioning) and creative leaps (for example, months after the Kathy shapes lesson, when the class was working on volume and surface area, one boy suddenly invented the notion of a *Kathy cube* – a three dimensional shape whose volume was numerically equal to its surface area, for which he produced an example).

Lessons which were regarded as didactic were based on some mathematical topic where, crucially, there were *definitions* to be given. Examples were vectors and trigonometry. This seemed to demand some form of exposition from the teacher – back to *telling*. It seemed that 'didactic' teaching was associated with giving definitions. In the words quoted above, the teacher had mentioned *probability* – 'this over this', a definition. He went on to say,

> That's nearly a definition isn't it? That is, I suppose that's one area I'm still sorting out in my own mind. Because things like $\overrightarrow{AB}$ and *vector* is a definition. What work do you do up to that definition?

I draw attention to the words "sorting out in my own mind", as evidence of this teacher's overt exploration of his own practice of teaching.

CLASSROOM INTERACTION

The answer to the teacher's question, above, seemed to lie in classroom interaction. I had probed further, choosing as a focus *Pythagoras' theorem*, an accepted part of the body of knowledge students were required to address. Here is an excerpt from the interview which should be seen in the context of a good relationship and mutual understanding between the teacher and myself:

BJ I'm going to push you by choosing an example. *Pythagoras* keeps popping up, and Pythagoras is something that you want all the kids in your group to know about. Now, in a sense there's some knowledge there that's referred to by the term 'Pythagoras'. And, I could pin you down even further to say what it is, you know, what *is* this thing called Pythagoras that you want them to know about?

Ben My kids have made a conjecture about Pythagoras which I agree with. So, it's not my knowledge. It's their knowledge.

BJ How did they come to that?

Ben Because I set up a set of activities leading in that direction.

BJ Right, now what if they'd never got to what you class as being Pythagoras? Is it important enough to pursue it in some other way if they never actually get there?

Ben Yeah.

BJ What other ways are there of doing that?

He laughed, paused, and then continued:

Ben You're talking in the abstract which then becomes difficult, aren't you now! Because you're not talking about particular classes or particular groups of students etc. Because I've always found in a group of students if I've given them an activity to lead somewhere there are some students who got there. It sounds horrible that. *Came up with a conjecture which is going to be useful for the future if I got there*, yes? And then you can start sharing it because students can then relate it to their experiences.

BJ So, it's alright for them to share with each other, but not alright for you to share with them?

Ben If I share with them I've got to be careful because I've got to share what I know *within those experiences*. (Jaworski, 1994)

Ben's words "You're talking in the abstract ..." highlight for me a difference between practice and a theoretical articulation of practice. He suggested that I was talking of generalities which were inappropriate to his situation, where the specifics mattered more. He struggled to express generally what he saw happening in practice, and was not happy with the words used. This is a clear example of the theory-practice dialectic, where our speaking *about* practice cannot capture the essence of the practice but only approximate to it. Nevertheless, as a researcher, I go on to present a theoretical account of the teacher's operation.

The teacher saw it as his task to create classroom experiences to provide opportunities for learning. Much of the work that I saw involved students in interactive groups. For example, in the *Kathy shapes* lesson, groups were formed by students deciding what shape they would work on initially. The group working on rectangles sought *Kathy rectangles*. Each person in the group worked on separate examples, then they pooled their findings. Subsequently, together, they developed a 'homing-in' process based on bisection of an interval on length and width. The *triangles* group got stuck finding areas of triangles. It emerged from discussion of an isosceles triangle with two sides of length 2 units, that several students thought the vertical height would be 2 or greater. The interactive nature of the work allowed knowledge to grow within the group (an example of intersubjectivity to which I shall return shortly). The teacher, listening and observing, offered support or ideas or challenges related to the students' experiences. In some cases he could "give them a bit of mathematics".

I put this in quotes, because it is precisely what the teacher claimed to do in interaction with one student in a lesson devoted to developing coursework for assessment purposes. The student had articulated a rule to describe a situation on which she was working, and Ben felt that she could refine her rule if she had access to the *distributive* law. He had therefore proceeded to explain this law to her – as he said, to 'give' her the distributive law (Jaworski, 1994a). Thus, he seemed to contradict his own statement, "giving this knowledge, sharing your knowledge with people, which is not possible". What he had done, fitted better with the later statement: "If I share with them, I've got to be careful, because I've got to share what I know *within those experiences*." Was he thus differentiating situations where experience seemed fruitful for 'sharing' of knowledge? Could articulation of such situations enhance teaching knowledge related to von Glasersfeld's theoretical statement: "language can be used as a tool in the process of guiding the student's construction"?

Research Synthesis: Theory and Practice

In my analysis of the work of this teacher, I interpreted his thinking initially in terms of a radical constructivist perspective. From what he said, I felt he was concerned for students to have opportunity to construct knowledge for themselves, and this seemed an individual focus. However, observing what he did, and coming back to some of his statements, I saw a strong *social* constructivist dimension to his thinking. This was not well articulated and he seemed to be struggling with his own understanding of the nature of mathematical knowledge and its classroom construction. However, as a researcher, I was no less struggling with my interpretation of classroom events and their theoretical rationalisation. The essence of reflective practice leading to knowledge construction involves such struggle (Dewey, 1933; von Glasersfeld, 1984: Kemmis, 1985; Schön 1987: Jaworski, 1994).

We are all seriously influenced by the cultural domains in which we live and act, and while we regard these uncritically, our thought processes may seem derivative (Vygotsky, 1979 cited in Wertsch & Toma, 1995). However, a constructivist perspective focuses on the challenges and constraints which force a critiquing, and subsequent adaptation of what we know. Thus, a social constructivist position is that knowledge in the social domain, is a negotiated synthesis of social and cultural practices, through the experience of individual knowers. This articulation seems more compatible with the Piagetian than the Vygotskian position.

Yet, in much of the discussion of classroom issues above, social and cultural influences on human thought are evident. I have pointed in particular to social influences of the cultures of mathematics, schooling, and everyday familiarity, but there are many others. Their influence must be part of a teacher's concern. In mathematics teaching particularly, where we draw on a variety of registers, meanings can only be clarified and understood by critical recognition of the social origins of words, phrases and symbolic structures. It is important, also, to be aware of the influence on mathematical cognition of social forces such as the dominance of western logicism and the secondary position of women, in order to avoid elitism or detrimental discrimination. It seems crucial to look critically at the contrasting metaphors of knowledge construction and enculturation. In our emphasis as constructivists on individual cognizing, we dangerously neglect sociocultural forces which constrain possibilities and promote outcomes. The statement from Taylor and Campbell Williams (1993), quoted above, speaks of acknowledging "the sociocultural and socioemotional contexts of learning". But, what does this mean in practical terms? What are the issues and dilemmas which such a position imposes on teachers? What are the consequences for teachers of avoiding such dilemmas? The classroom situations discussed may be seen as starting points for further exploration of these questions.

Finally, I must return to the issue of how knowledge grows within an interactive environment. Taylor and Campbell Williams (op cit) speak of "the learner as an interactive co-constructor of knowledge". What is involved in co-construction, and what is the nature of intersubjectivity? My example, above, of students jointly constructing a means of locating *Kathy rectangles*, suggests that intersubjective knowledge exists in some way within the group, separate from the individual members of the group. It seems clear to me, however, that whatever it may seem, this is actually not the case. As an observer, I have my own interpretation of such events, which I can support in terms of the words of individuals, which I can compare and contrast with interpretations of the teacher and possibly students, and which I can offer for critical consideration by members of a wider educational community. Ultimately, however, all I have is my own cognition, and this is true for each member of the group. Each person's 'coming to know' within the group is influenced by contribution of others in the group. In a rapid exchange of views it is hard to track influences on cognition, or indeed to know what any individual makes of the concepts being negotiated. The observer (teacher, perhaps), taking a more distant viewpoint, may gain a more global sense of the totality of the contributions than any individual involved, but it is dangerous therefore to assume that any individual would perceive the same totality.

The Purdue team, Cobb, Yackel and Wood, took as central to their teaching experiment in second grade mathematics lessons an analysis of processes of negotiation and sharing of meaning in the construction of classroom mathematics (see, for example, Wood *et al*, 1993). Their interactionist approach to analysis was designed to highlight relationships between social interaction and the construction of mathematics within the group. It was possible to look at conversations in Ben's classroom and subject them to a similar form of interactionist analysis to trace knowledge growth. Ben's words above, "My kids have made a conjecture that I agree with" seem to be cast in intersubjective mode. They reflect a perspective of developing shared meanings. From this perspective, it makes sense for the teacher to tell or explain within an understanding of the social context and experience of the students.

Maturana and Varela (1987) make clear that there is no difference, biologically, between an organism learning from its environment and learning from other organisms. Such learning results in the creation of new structures. In a social environment a human learner is challenged by other individuals who have a powerful role to play. Through use of language and social interchange, individual knowledge can be challenged and new knowledge constructed. Moreover, there can grow within the environment something shared by individuals within it which might be referred to as *common*, or *intersubjective*, knowledge. This was what I meant when I said above that 'the interactive nature of the work allowed knowledge to grow within the group'.

The didactic maze

For mathematics teachers there is a dialectical relationship between what they want students to know or to learn and creation of classroom processes by which such learning may be achieved. Implicit in the tensions which arise is an involvement in and awareness of influencing cultures. The extent of this influence is hard to judge, especially by people who are themselves uncritically culture bound. Each teacher has responsibility to teach mathematics and to deliver the mathematical curriculum: not necessarily the same task. The teachers' own perceptions of mathematics and of learning are central to their classroom approach. Curriculum statements identify mathematical concepts which the learner is required to know. These include items of knowledge such as Pythagoras' theorem, and 'all quadrilaterals tessellate'. Such knowledge will be tested by standardised tests or examinations which will require standard answers. These requirements fit more closely with an objectivist paradigm of knowledge transfer than with one of knowledge construction. They encourage an ontological commitment to the curriculum items, or a cultural absorption, to use an alternative paradigm. The teacher working from a constructivist perspective is thus led into a position of having certain knowledge to inculcate or elicit, while recognising that such knowledge is relative to individual experience for each student. The sharing of perceptions through articulation, listening and negotiation allows growth of intersubjective knowledge through which perspectives can be challenged. In particular the teacher can offer explanations as a part of the interactive discourse. These explanations need to be negotiated along with all other statements. However, unequal power positions might mean that the teacher's statements are not challenged and students believe because they are told to believe rather than because the ideas make sense in their worlds. Issues of social justice and cultural discrimination add to the didactic maze.

In conclusion

I hope that this paper has highlighted a number of worlds or discourses, three of these being: the academic world of contrasting theoretical positions aiming to inform the educational context; the practical world of students' everyday lives, classroom relationships, and teaching decisions; and the research world which tries to act as a bridge between the other two, making sense of each and telling stories about potential links, in a desire to illuminate both. I agree with Gergen (1995), who emphasises that "there is no means by which practical derivatives can simply be squeezed from a theory of knowledge". As he says, "theories can specify neither the particulars to which they must be applied nor the contexts in which they may be rendered intelligible. There are no actions that follow necessarily from a given theory" (1995, p29). However, I know that considerations of theory have been central to my own conceptualisations of mathematics learning and teaching, however

imperfect these remain. As in the classrooms described by Cobb, Wood and Yackel, and in many of the ones in which I have participated, *negotiation* is more than a device to ensure 'correct' knowledge construction, it is a dialogic mode which creates a discourse from which individuals make their own sense and communicate with others. This dialogic mode in theory generation seems potentially its most valuable attribute. Perhaps whether we find a unifying theory to link constructivism and socioculturalism is not the most important consideration. What is most valuable is the ongoing debate through which we develop awarenesses of educational means and practice and consequentially perhaps a more effective process of human development.

References

Bauersfeld, H. (1994) 'Theoretical Perspectives on Interaction in the Mathematics Classroom', in Biehler R. et al (eds) *The Didactics of Mathematics as a Scientific Discipline*, Dordrecht: Kluwer

Bruner, J.S. (1985) 'Vygotsky, A Historial and Conceptual Perspective', in Wertsch, J.V. (Ed) *Culture, Communication and Cognition: Vygotskian Perpsectives*. Cambridge: Cambridge University Press.

Bruner, J.S. (1986) *Actual Minds, Possible Worlds*. London: Harvard University Press

Burton, L. (1996) 'The Implications of a Narrative Approach for the Learning of Mathematics', paper presented at the *Conference of the Growing Mind*, in the Symposium *Learning Mathematics: From Hierarchies to Networks*. Geneva, Sepetember, 1996

Cobb, P. Wood, T. and Yackel E. (1991) 'A constructivist approach to second grade mathematics', in E. von Glasersfeld (ed.) Radical constructivism in mathematics education. The Netherlands: Reidel

Confrey, J. (1991) 'Steering a Course Between Vygotsky and Piaget', *Educational Researcher* 20(2) 29-32

Confrey, J. (1995) 'How Compatible are Radical Constructivism, Sociocultural Approaches, and Social Constructivism?' in Steffe, L.P. & Gale, J. (Eds) *Constructivism in Education*. Hove, UK: Lawrence Erlbaum Associates

Davis, R.B., Maher, C.A. and Noddings, N. (1990) (Eds) Constructivist Views on the Learning and Teaching of Mathematics. Journal for Research in Mathematics Education, Monograph Number 4. Reston, Virginia: National Council of Teachers of Mathematics

Dewey, J. (1933)*How we Think*. London: D.C. Heath & Co.

Driver, R. (1983) The pupil as scientist. Milton Keynes: Open University Press

Edwards D. and Mercer N. (1987) *Common Knowledge*. London: Methuen

Ernest, P. (1991) *The Philosophy of Mathematics Education*. London: Falmer Press

Gergen, K.J. (1995) 'Social Construction and the Educational Process', in Steffe, L.P. & Gale, J. (Eds) *Constructivism in Education*. Hove, UK: Lawrence Erlbaum Associates

Hammersley, M. (1993) 'On Constructivism and Educational Research Methodology', *PME News*, May. Oxford: University of Oxford

Jaworski, B. (1994) *Investigating Mathematics Teaching: A Constructivist Enquiry*. London: Falmer Press

Jaworski, B. (1994a) 'The Social Construction of Classroom Knowledge' in Ponte, J.P. and Matos, J.F. (eds) *Proceedings of the 18th International Conference for the Psychology of Mathematics Education*. Lisbon: University of Lisbon

Kemmis, S. (1985) 'Action Research and the Politics of Reflection' in Boud, D. et al (Eds) *Reflection: Turning Experience into Learning*. London: Kogan Page

Lave, J. & Wenger, E. (1991) *Situated Learning: Legitimate Peripheral Participation*. Cambridge, MA: Cambridge University Press

Lerman, S. (1989) 'Constructivism, Mathematics and Mathematics Education', *Educational Studies in Mathematics*, 20, The Netherlands: Kluwer

Lerman, S. (1996) 'Intersubjectivity in Mathematics Learning: A Challenge to the Radical Constructivist Paradigm' *Journal for Research in Mathematics Education*. 27 2. Reston, Virginia: NCTM

Lerman S. (1996a) Review of Jaworski, B. (1994 op cit) Karlsruhe, Germany: ZDM

Mason, J. (1994) *Researching from the inside in Mathematics Education*. Milton Keynes: Centre for Mathematics Education, Open University

Maturana and Varela (1987) *The Tree of Knowledge*. Boston: Shambala

Noddings, N. (1990) 'Constructivism in Mathematics Teaching', in Davis, R.B., Maher, C.A. and Noddings, N.(Eds) *Constructivist Views on the Learning and Teaching of Mathematics*. Journal for Research in Mathematics Education, Monograph Number 4. Reston, Virginia: National Council of Teachers of Mathematics

Piaget, J. (1950) *The Psychology of Intelligence*. London: Routledge and Kegan Paul

Schön, D.A. (1987) *Educating the Reflective Practitioner*. Oxford: Jossey Bass

Shotter, J. (1995) 'In Dialogue: Social Constructionism and Radical Constructivism', in Steffe, L.P. & Gale, J. (Eds) *Constructivism in Education*. Hove, UK: Lawrence Erlbaum Associates

Taylor, P. and Campbell-Williams, M. 'Discourse towards balanced rationality in the high school mathematics classroom: Ideas from Habermas's critical theory', in Taylor, P.C.S. and Malone, A.J. (Eds) (1993) *Constructivist Interpretations of Teaching and Learning Mathematics*. Perth, Australia: Curtin University of Technology

von Glasersfeld E. (1982) 'An interpretation of Piaget's Constructivism', *Revue Internationale de Philosophie* 142-4 pp612-635

von Glasersfeld E. (1983) 'Learning as a Constructive Activity'. In PME-NA Proceedings, Montreal, September-October,1983, 41-69

von Glasersfeld E. (1984) 'An introduction to radical constructivism'. In P. Watzlawick (ed.) The invented reality. London: W.W. Naughton & Co.

von Glasersfeld E. (1987) 'Constructivism' in Husen, T. & Postlethwaite (Eds) *International Encyclopaedia of Education*, Supplement Vol 1, Oxford: Pergammon

von Glasersfeld, E. (1990) 'An Exposition of Constructivism: Why Some like it Radical', in Davis, R.B., Maher, C.A. and Noddings, N. (Eds) *Constructivist Views on the Learning and Teachinbg of Mathematics*. Journal for Research in Mathematics Education, Monograph Number 4. Reston, Virginia: National Council of Teachers of Mathematics

Vygotsky, L. (1978) *Mind in Society: The Development of the Higher Psychological Processes*. London: Harvard University Press

Vygotsky, L. (1981) 'The Genesis of Higher Mental Functions' in Wertsch, J.V. (Ed) *The Concept of Activity in Soviet Psychology*. Armonk: Sharpe

Wertsch, J.V. & Toma, C. (1995) 'Discourse and Learning in the Classroom: A Sociocultural Approach', in Steffe, L.P. & Gale, J. (Eds) *Constructivism in Education*. Hove, UK: Lawrence Erlbaum Associates

Wertsch, J.V. (1991) *Voices of the Mind: A Sociocultural Approach to Meditated Action. Cambridge, MA: Cambridge*

Wheeler, D. (1982) 'Mathematization matters', *For the Learning of Mathematics*, 3, 1, 45-47

Wood, T., Cobb, P., Yackel, E. and Dillon, D. (1993) (Eds.) 'Rethinking Elementary School Mathematics : Insights and Issues', *Journal for Research in Mathematics Education, Monograph number 6*. Reston Virginia: National Council of Teachers of Mathematics

For further discussion on this paper, please contact:

Dr. Barbara Jaworski
Centre for Mathematics Education Research
University of Oxford
Department of Educational Studies
15 Norham Gardens
Oxford OX2 6PY, UK

Phone:+44 1865 274024
Fax +44 1865 274027
E-mail: barbara.jaworski@edstud.ox.ac.uk

The paper is due to be published as a chapter in the book:
Learning Mathematics: From Hierarchies to Networks,
edited by Professor Leone Burton at the University of Birmingham, UK